# SIMULATING ARBITRARY-GEOMETRY ULTRASOUND TRANSDUCERS USING TRIANGLES

Jørgen Arendt Jensen

Department of Information Technology, Build. 344,
Technical University of Denmark, DK-2800 Lyngby, Denmark

## Abstract

Calculation of ultrasound fields from medical transducers is often done by applying linear acoustics and using the Tupholme-Stepanishen method of calculation. Here the spatial impulse response is found and, together with the basic one-dimensional pulse, it is used to find both the emitted and pulse-echo field. The spatial impulse response has only been determined analytically for a few geometries and using apodization over the transducer surface generally makes it impossible to find the response analytically. A popular approach to find the general field is thus to split the aperture into small rectangles, and then sum the weighted response from each of these. The problem with rectangles is their poor fit to apertures which do not have straight edges, such as circular and oval shapes. The simulation thus introduces artifacts in the response, that necessitates the use of a large number of rectangles for a precise simulation.

A triangle better fits these aperture shapes, and the field from a triangle has recently been derived [1]. A new field simulation program has been made based on the triangular shape. It is written in C and interfaced to the Matlab environment through a set of M-files. A large number of transducers can be defined and their properties manipulated. The program can calculate all types of ultrasound fields, and can also be used for simulating B-mode and color flow images. Both the focusing and apodization can be set to be dynamic with respect to time, and it is thus possible to simulate images focused at different zones. The time-integrated spatial impulse response is used in the program to minimize the effect of the sharp edges of the spatial impulse response in a sampled signal. Since the integrated response from a triangular element cannot be analytically evaluated, a simple numerical integration is used.

Using this program, the geometrical artifacts from fitting the aperture with a basic element are significantly reduced and is in most instances negligible. The time for running the program is, however, increased by a factor of 3.3 to 1000 compared to using the simple far-field response of a rectangle, as the triangle equations are far more complicated. This approach is therefore best suited for accurate modeling of fields, whereas the rectangle program is better suited to make fast simulated images, since contributions from many scatterers are summed here and the error is thereby reduced.

## 1 Introduction

The optimization of fields from ultrasound transducers and thereby image quality is most conveniently performed by using computer simulations. The model most often preferred for the calculation of the field is that developed by Tupholme and Stepanishen [2], [3], [4]. Here the spatial impulse response is calculated, and it describes the field at a particular point in space, when the transducer is excited with a delta impulse. The approach is based on the Rayleigh integral, and relies on linear propagation of the emitted sound. The model is very general and can be used for calculating the field for flat and slightly curved transducers in pulsed and continuous wave mode, and can yield both the emitted and pulse-echo field from small scatterers [5]. One drawback of the approach is, however, that solutions are difficult to derive for many transducer geometries, and that apodization also is difficult to incorporate.

One possible method for alleviating this problem is to divide the transducer surface into smaller areas with a known field, and then sum the contributions from all the elementary areas. This has been done by using rectangles [6], and a far field approximation for the field from the rectangles. The approach is very fast, but has the problem that fitting rectangles to a curved radiator necessitates the use of many rectangles for a good fit. A much better fit is obtained with a triangle, but it is only recently that the field from a triangle has been known [1]. This paper describes a field simulation program based on the newly developed triangle solution.

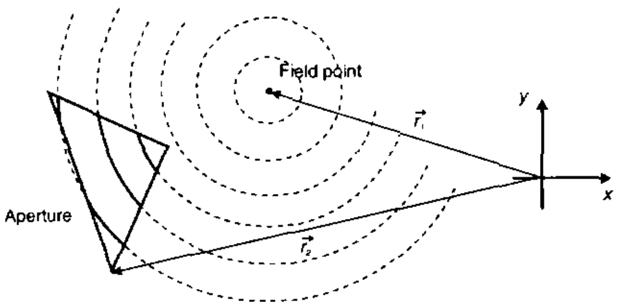


Figure 1: Intersection of spherical waves from the field point by the triangle, when the field point is projected onto the plane of the triangle.

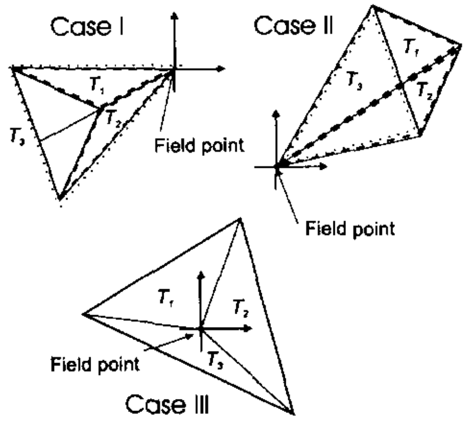


Figure 2: Triangles used for calculating the spatial impulse response.

## 2 Simulation using triangles

The spatial impulse response is found from the integral given by:

$$h(\vec{r}_1, t) = \int_S \frac{\delta(t - \frac{|\vec{r}_1 - \vec{r}_2|}{c})}{2\pi \mid \vec{r}_1 - \vec{r}_2 \mid} dS \qquad (1)$$

It is essentially a statement of Huyghen's principle that the field is found by summing the radiated spherical waves from all parts of the aperture. This can also be reformulated, due to acoustic reciprocity, as finding the part of the spherical wave emanating from the field point that intersects the aperture. The problem is then reduced to projecting the field point onto the plane coinciding with the triangle, and then finding the intersection of the circle with the triangle as shown in Fig. 1. The intersections then depend on the orientation of the triangle, and the calculation is eased by introducing three triangles, that have one edge point at the field point. Three different cases then arise as shown in Fig. 2, and the fields are given by:

| Case | |
|---|---|
| I | $h(\vec{r}_1, t) = h_{T_3}(\vec{r}_1, t) - h_{T_1}(\vec{r}_1, t) - h_{T_2}(\vec{r}_1, t)$ |
| II | $h(\vec{r}_1, t) = h_{T_1}(\vec{r}_1, t) + h_{T_2}(\vec{r}_1, t) - h_{T_3}(\vec{r}_1, t)$ |
| III | $h(\vec{r}_1, t) = h_{T_1}(\vec{r}_1, t) + h_{T_2}(\vec{r}_1, t) + h_{T_3}(\vec{r}_1, t)$ |

The calculation of the field is then reduced to merely finding the intersections with the circle and the triangle. Five different cases exist, as shown in [1].

The spatial impulse response calculated has discontinuities, when the projected spherical wave intersect the edges of the triangles. This creates problems in a sampled system, since the response has components for infinitely high frequencies. The spatial impulse response will, however, be convolved with the ultrasound pulse, and is thereby band-limited. It is thus not the high frequency components that are very important, but the actual energy of the response. A better accuracy can be attained by using the integrated spatial impulse response to account for all the energy. This is done in the program through a simple trapezoidal integration followed by a digital differentiation. This significantly lowers the demand on the sampling frequency, and is equal to other authors' increase of sampling frequency at discontinuities. Here the discontinuities are, however, explicitly taken into account, and the preservation of the energy in the response is ensured.

It should be noted that the solution used here is exact, and that no approximation in calculating the field for flat transducers is introduced. The only source of error is, thus, the aliasing introduced by the finite sampling frequency.

## 3 The FIELD program

The triangle solution described here is an extension of the FIELD program that has been developed and improved since 1991. The original program was based on a menu interface, and could readily be used without much experience. The limitation for this version was that time-varying focusing and apodization could not be implemented easily, and therefore ultrasound imaging was not easy to simulate with this program. A FIELD II version was developed with a very close interface to the Matlab environment to address this problem. Procedures for describing transducers and doing field calculations are then directly called from Matlab and it is possible to have both time-dependent focusing and apodization. Using the Matlab environment, it is possible to make field simulation scripts, and this has made it possible to directly simulate ultrasound imaging in software. Such imaging is described in [7].

The current version of the program consists of roughly 5500 lines of standard ANSI C. About 1500 of these lines of code is used for the triangle solution. The program has been successfully compiled and run on all major workstations and the PC. The major features of the program are:

- Transducer modeled by dividing it into rectangles and triangles.
- C program interfaced to Matlab.
- Matlab used as front end.
- Can handle any transducer geometry.
- Physical understanding of transducer.
- Pre-defined types: linear and phased arrays, 2D arrays, round plane and concave apertures.
- Any focusing, apodization, and excitation pulse can be used.
- Time-dependent focusing and apodization.
- Can calculate all types of fields (emitted, received, pulsed, CW)
- Can generate artificial ultrasound images (phased and linear array images with multiple receive and transmit foci).
- Data storage not necessary.
- Postprocessing in Matlab
- Versions for: DEC Station (MIPS, ALPHA), HP, SUN, SGI and PC (should be portable to all computers, where Matlab is running).

The program can be found at the web site: http://www.it.dtu.dk/~jaj/field/field.html, where the executable code freely can be downloaded.

# 4 Performance

The quantity affecting the accuracy of the approach is the sampling frequency, as aliasing will take place due to the sharp discontinuities of the spatial impulse responses. The effect of this on the pulse-echo response received from a collection of scatterers is shown in Fig. 3. Here a 32-element linear-array transducer is used. Each element has a width 0.15 mm, a kerf of 0.05 mm, and the height is 10 mm. Each physical element is divided into two triangles. The focus is 60 mm from the face of the aperture. A random collection of 200 scatterers with a Gaussian distribution of amplitudes within in a box of 10 by 5 by 5 mm placed 30 mm from the center of the transducer is used as the target. The calculated response at 2 GHz is compared with responses from other sampling frequencies. It can be seen that using sampling frequencies below 100 MHz is not advisable, and a 10% accuracy is attained at 300 MHz. This might seem like a high error, but quite precise results for the point spread function can be attained even at moderate sampling frequencies, as shown in

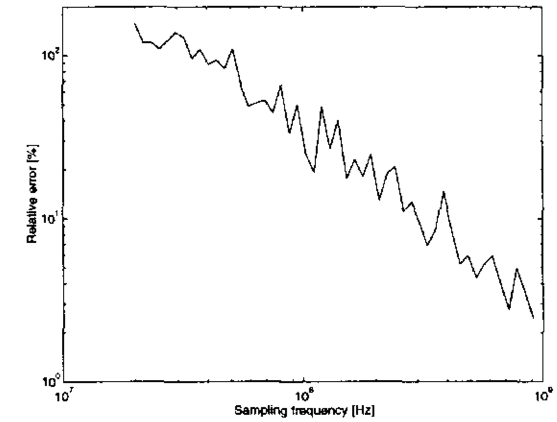


Figure 3: Error of calculated response as a function of sampling frequency

the next figure. This is for a 64-element linear-array trans-

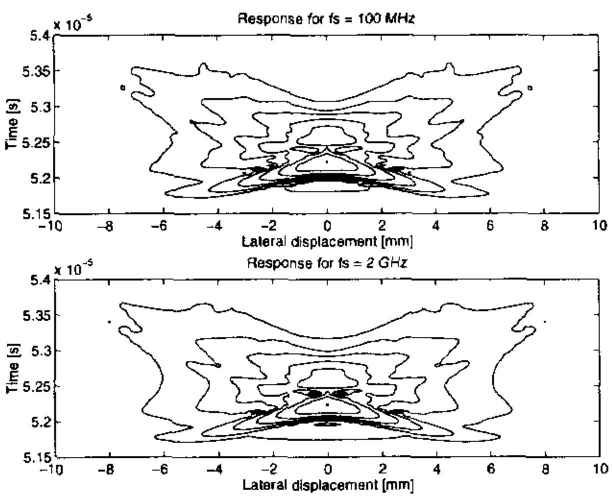


Figure 4: Point spread function of 64 element linear array calculated for different sampling frequencies. There is 6 dB between the contour lines.

ducer excited by a 3 cycle 5 MHz pulse, and the envelope of the pulse-echo response is shown on a logarithmic scale. A sampling frequency of 100 MHz and 2 GHz are used, and nearly identical responses down to -36 dB are seen.

Fig. 5 shows a comparison between using rectangles and triangles in the simulation. The top graph shows the attainable accuracy when using a different number of rectangles. A square transducer of 4 × 4 mm was used, and the response calculated was compared to the correct response for the given sampling frequency. The pulse-echo field is calculated, and

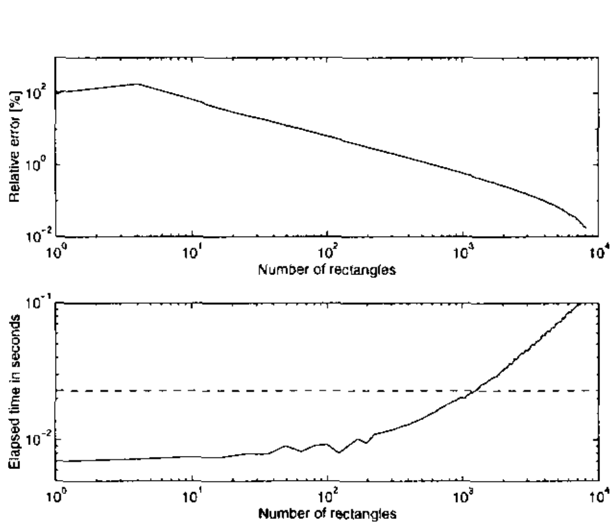


Figure 5: Error of calculated response from triangles and calculation times. The horizontal line in the bottom graph indicates the calculation time when using the triangle solution.

the target is a collection of 2000 scatterers with Gaussian amplitudes and a random placement within a box $20 \times 10 \times 30$ mm starting 20 mm from the front face of the transducer. It is seen that the accuracy is increased with the number of rectangles. The lower graph shows the calculation time per scatterer for different number of rectangles. The horizontal line indicates the calculation time for the triangle solution. The times were found on a 200 MHz Pentium Pro PC with 32 MBytes RAM. The elapsed time is 0.024 seconds for a pair of triangles and 0.0073 seconds for one rectangle, making the rectangle calculation 3.3 times faster. The rectangle calculation, however, has a fairly large overhead, and only increases slowly with the number of rectangles. It is thus possible to use more than 1000 rectangles for one pair of triangles, and thereby attain an accuracy of under 1%. The advantage of using triangles is that the solution always is correct regardless of the distance to the aperture. Further, the memory requirements are much lower for one pair of triangles than 1000 rectangles, which gives a significant advantage for solving large, high precision problems.

The choice between using triangles or rectangles is, thus, dependent on the application. Calculating images using array transducers with small elements is most conveniently done by using rectangles, whereas high precision imaging with large apertures is more conviniently done using the accurate triangles.

Several possibilities also exists for improving the speed of the triangle solution. Firstly, the theoretical solution is somewhat cumbersome, as the field from three triangles are calculated. Other more efficient solutions might be found. Secondly, a numerical integration of the responses are performed. A significant speed increase could be obtained, if a closed form solution to the equation:

$$\begin{aligned} \theta_{\text{int}}(t) &= \int_{t_1}^{t_2} \arccos\left(\frac{k_2 + \sqrt{k_1 r^2(t) - k_3}}{k_4 r(t)}\right) dt \\ r(t) &= \sqrt{(ct)^2 - z^2} \end{aligned} \quad (2)$$

could be found. Here $c$ is the speed of sound and $z, k_1, k_2, k_3, k_4$ are constants. Thirdly, it is quite possible that a further optimization of the code might increase the performance.

# References

[1] J. A. Jensen. Ultrasound fields from triangular apertures. *J. Acoust. Soc. Am.*, 1996a.

[2] G. E. Tupholme. Generation of acoustic pulses by baffled plane pistons. *Mathematika*, 16:209–224, 1969.

[3] P. R. Stepanishen. Transient radiation from pistons in an infinte planar baffle. *J. Acoust. Soc. Am.*, 49:1629–1638, 1971.

[4] P. R. Stepanishen. The time-dependent force and radiation impedance on a piston in a rigid infinite planar baffle. *J. Acoust. Soc. Am.*, 49:841–849, 1971.

[5] J. A. Jensen. A model for the propagation and scattering of ultrasound in tissue. *J. Acoust. Soc. Am.*, 89:182–191, 1991.

[6] J. A. Jensen and N. B. Svendsen. Calculation of pressure fields from arbitrarily shaped, apodized, and excited ultrasound transducers. *IEEE Trans. Ultrason., Ferroelec., Freq. Contr.*, 39:262–267, 1992.

[7] J. A. Jensen. Field: A program for simulating ultrasound systems. *Med. Biol. Eng. Comp.*, 10th Nordic-Baltic Conference on Biomedical Imaging, Vol. 4, Supplement 1, Part 1:351–353, 1996b.